New Jersey Department of Education

# Problem Solving in K–3 Mathematics

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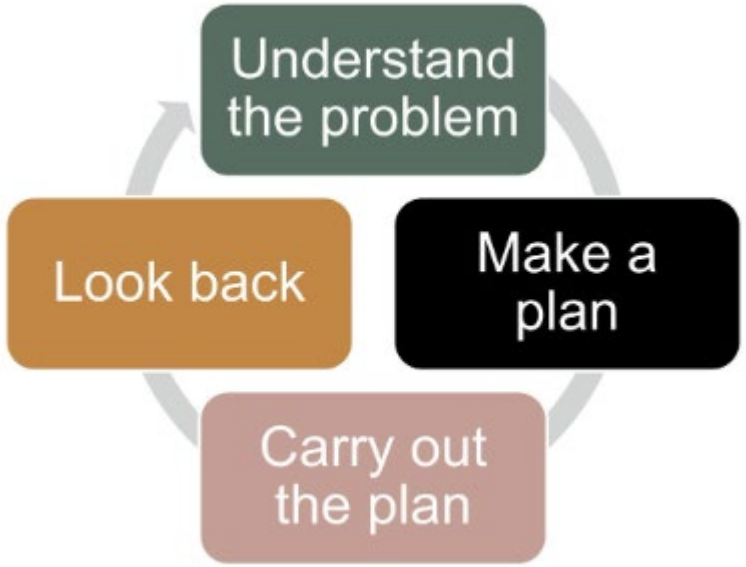
# Introduction

Problem Solving vs. Word Problems

Problem solving is about thinking, reasoning, exploring, hypothesizing, noticing, and wondering. Problem Solving takes dedication and perseverance. There are many ways that problem solving occurs…word problems are one way.

George Polya is often called the Father of Problem Solving. He wrote a book called How to Solve It in 1945 and outlined a 4-step process for solving problems.



# Agenda

- Standards
- Problem Types
- Schema Based Instruction
- Sense making, not tricks
- Tools, Resources, and Rich Tasks
- Productive Struggle
- Questions or Comments

# Standards 1 of 3

PS 4.2.2

Begin to represent simple word problem data in pictures and drawings.

K.OA.A.2

Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

1.OA.A.1

Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.*

1.OA.A.2

Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

2.OA.A.1

Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.*

# Standards 2 of 3

3.OA.A.3.

Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

3.OA.D.8

Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding

3.OA.A.3

Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem

# Standards 2 of 2

The state standards use a schema-based framework for story problems, organizing them by type. This classification scheme for addition and subtraction illustrates the distinctions among the different types of word problems.

Schema-based instruction (SBI) is an evidence-based practice used in math that supports the underlying structure of any given word problem to find solutions to mathematical problems and apply that knowledge to future problems. It focuses on the actions or relationships in the problem.

# Problem Types 1 of 4

| Problem Type | Result Unknown | Change Unknown | Start Unknown |
|---|---|---|---|
| **Add to** (Join Problem Types) | 1. Kindergarten subtype Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ? | 2. Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5 | **3. Difficult subtype** Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 =5 |
| **Take From** (Separate Problems) | 4. Kindergarten subtype Five apples were on the table. I ate two apples. How many apples are on the table now? 5–2 = ? | 5. Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 –? = 3 | **6. Difficult subtype** Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3 |

# Problem Types 2 of 4

## Problem Type: Put Together Take Apart

| Total (or Whole) Unknown | Addend (or Part ) Unknown | Both Addends (or Parts) Unknown |
|---|---|---|
| 7. Kindergarten subtype<br>Three red apples and two green apples are on the table. How many apples are on the table?<br>3 + 2 = ? | **8. Kindergarten subtype**<br>Five apples are on the table. Three are red and the rest are green. How many apples are green?<br>3 + ? = 5<br>5–3 = ? | 9. Grandma has five flowers. How many can she put in the pink vase and how many in her blue vase?<br>5 = 0 + 5<br>5 + 0<br>5 = 1 +4<br>5 = 4 +1<br>5 = 2 + 3<br>5 = 3 + 2 |

# Problem Types 3 of 4

## Problem Type: Compare

| Difference Unknown | Bigger Unknown | Smaller Unknown |
|---|---|---|
| 10. ("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? | 11. (Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? | 12. (Version with "more"):Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? |
| 13. ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have then Julie?<br>2 + ? = 5<br>5 − 2 = ? | 14. (Version with "fewer") **Difficult subtype**: Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have?2 + 3 = ?<br>3 + 2 = ? | 15. (Version with "fewer") **Difficult subtype**: Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have?<br>5 – 3 = ?<br>? + 3 = 5 |

# Problem Types 4 of 4

| Problem Type | Unknown Product | Unknown Product Group Size Unknown ("How many in each group?" Division) | Number of Groups Unknown ("How many groups?" Division) |
|---|---|---|---|
| **Equal Groups** | There are 3 bags with 6 plums in each bag. How many plums are there in all?<br><br>Measurement example. You need 3 lengths of string, each 6 inches long. How much string will you need altogether? | If 18 plums are shared equally into 3 bags, then how many plums will be in each bag?<br><br>Measurement example. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be? | If 18 plums are to be packed 6 to a bag, then how many bags are needed?<br><br>Measurement example. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have? |
| **Arrays, Area** | There are 3 rows of apples with 6 apples in each row. How many apples are there?<br><br>Area example. What is the area of a 3 cm by 6 cm rectangle? | If 18 apples are arranged into 3 equal rows, how many apples will be in each row?<br><br>Area example. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it? | If 18 apples are arranged into equal rows of 6 apples, how many rows will there be?<br><br>Area example. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it? |

# Schema Based Instruction 1 of 7

## Join Problem Types

Join problems involve a direct or implied action in which a set is increased by a particular amount. There are three distinct types of join problems which vary based on the which quantity is unknown.

**Result Unknown**:

Robin had 6 toy cars. Her friend gave her 8 more toy cars. How many cars did she have then?

**Change Unknown**:

Robin had 6 toy cards. Her friend gave her some more toy cars and then she had 14 cars. How many did her friend give her?

**Start Unknown**:

Robin had some toy cars. Her friend gave her 8 more for her birthday. Then she had 14. How many did she have before her birthday?

# Schema Based Instruction 2 of 7

## Separate Problem Types

Similar to join problems, there is an action which takes place, but in these problems, the action in the problem is one in which the initial quantity is decreased rather than increased. There are three distinct types of Separate problems that are generated by varying the unknown.

**Result Unknown**:

Tonia had 13 pencils. She gave 4 pencils to Wayne. How many pencils does she have left?

**Change Unknown**:

Tonia had 13 pencils. She gave some to Wayne, then she had 9 pencils left. How many pencils did Tonia give to Wayne?

**Start Unknown**:

Tonia had some pencils. She gave 4 pencils to Wayne and then she had 9 pencils left. How many pencils did Tonia have to start with?

# Schema Based Instruction 3 of 7

## Part-Part-Whole Problem Types

There is no direct action or change over time in Part-Part-Whole problems. The problem involves static relationships among a set and its subsets.

**Whole Unknown**:

8 boys and 7 girls were playing soccer. How many children were playing soccer?

**Part Unknown**:

15 children were playing soccer. 8 were boys and the rest were girls. How many girls were playing soccer?

# Schema Based Instruction 4 of 7

## Compare Problem Types

Like Part-Part-Whole problems, there is a relationship between quantities rather than the action of joining or separating. They compare two distinct sets, rather than the relationship between a set and its subsets. In these problems there is a **referent** set (Joy has 8 books), a **compared** set (Greg has 12 books) and a **difference** (Greg has 4 more than Joy).

**Difference Unknown**:

Joy has 8 books. Greg has 12 books. How many more books does Greg have than Joy?

**Compared Set Unknown**:

Joy has 8 books. Greg has 4 more books than Joy. How many books does Greg have?

**Referent Unknown**:

Greg has 12 books. He has 4 more books than Joy. How many books does Joy have?

# Schema Based Instruction 5 of 7

Wording Matters! Children's ability to solve word problems depends mostly on their ability to understand and model the situation in the problem. Variations in wording of a problem can make the problem more, or less, difficult to understand, which teachers should consider.

Problems are easier to understand if they correspond to the action sequence:

**Jen just ate 3 cookies. She started with 9 cookies. How many does Jen have now?**

**Jen had 9 cookies. She ate 3 of them. How many does Jen have left?**

Reword problems so they make sense to children:

**There are 5 birds and there are 3 worms. How many more birds are there than worms?**

**There are 5 birds and 3 worms. How many birds won't get a worm?**

# Schema Based Instruction 6 of 7

Number sentences can be easily used to represent certain problems, particularly in Join and Separate problems. The three terms in the examples given, 6 + 8 = 14 and 13 – 4 = 9, correspond to the three quantities and are flexible.

In Part-Part-Whole problems and Compare problems, there is not such a clear correspondence because there is no action in the problem so no clear starting quantity, change quantity, or resulting quantity.

# Schema Based Instruction 7 of 7

Schema Based problem solving is a framework that is used to teach word problems (Carpenter et al., 2014). There are four basic types of addition and subtraction word problems. By varying the unknown, a total of eleven distinct types of word problems can be constructed.

When teachers understand the types of word problems and the hierarchy of difficulty, teachers can reflect about their students' thinking and professional discussions and planning amongst educators can occur.

These problem type names and categories are not for students to use and learn…this would be counterproductive. Students simply solve them naturally using the context of the problem. Students will start to see patterns in problems and recognize types naturally.

# Key Words 1 of 10

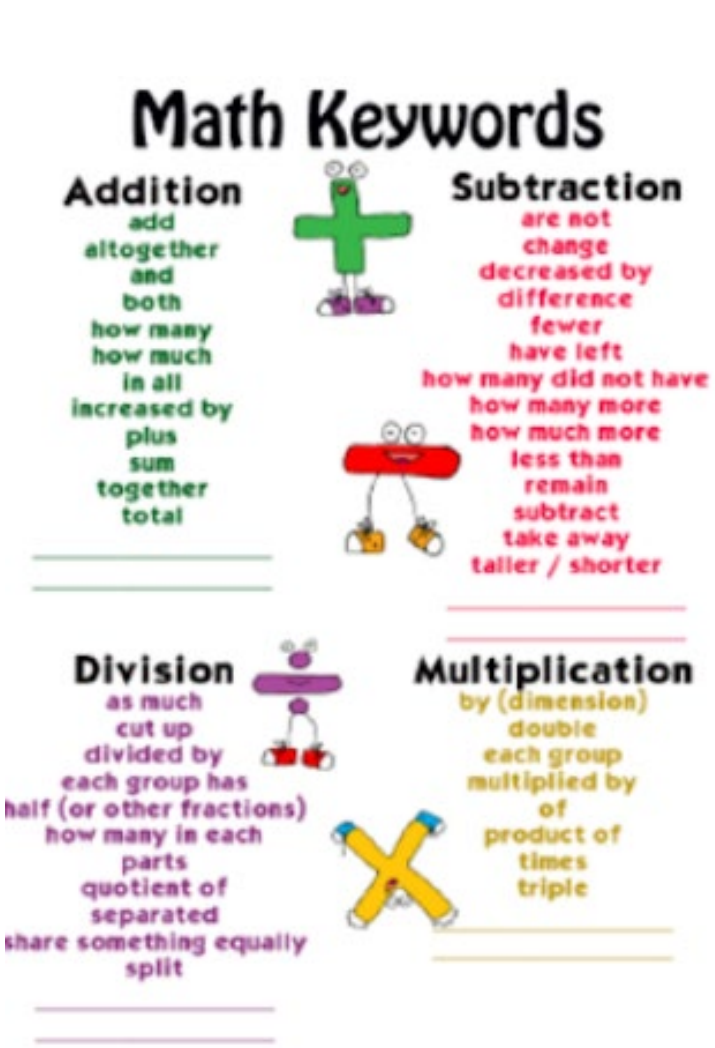
Standard for Mathematical Practice 1:

Make sense of problems and persevere in solving them.

Students who ‘make sense of problems’ should be able to:

- explain the meaning of a problem.
- plan a solution pathway rather than simply jumping to a solution attempt.
- continually ask themselves ‘Does this make sense?’.

# Key Words 2 of 10





# Text Version of Slide 20 (Math Keywords Poster)

## Addition

- Add
- Altogether
- And
- Both
- How many
- How much
- In all
- Increased by
- Plus
- Sum
- Together
- total

## Subtraction

- Are not
- Change
- Decreased by
- Difference
- Fewer
- Have left
- How many did not have
- How many more
- How much more
- Less than
- Remain
- Subtract
- Take away
- Taller/shorter

## Multiplication

- By (dimensions)
- Double
- Each group
- Multiplied by
- Of
- Product of
- Times
- Triple

## Division

- As much
- Cut up
- Divided by
- Each group has
- Half (or other fractions
- how many in each
- Parts
- Quotient of
- Separated
- Share something equally
- Split

# Text Version of Slide 20 (Middle Poster)

| + | − | × | ÷ |
|---|---|---|---|
| • Add | • Subtract | • Multiply | • Divide |
| • Total | • Fewer | • Times | • Each |
| • Sum | • How many more | • Every/each | • Quotient |
| • All together | • Left | • At this rate | • Per |
| • In all | • Less than | • Product | • Out of |
| • Both | • Minus | • Area | • Into |
| • Plus | • Need to | • Of | • Separated |
| • Combined | • Remain | | |
| | • Take away | | |

# Text Version of Slide 20 (Right Poster)

| + | − | × | ÷ |
|---|---|---|---|
| • Combined | • Subtract | • Triple | • Average |
| • Addition | • Decrease | • Twice | • Division |
| • In all | • Fewer | • Factor | • Split |
| • Total | • Remain | • Product | • Quotient |
| • Sum | • Take away | • Multiply | • Equal groups |
| • Together | • Minus | • Each | • Divide |
| • Plus | • Less than | • Per | • Half |
| • Both | • How many more | • In all | • Shared equally |
| • Perimeter | | • Multiple | • Each |
| • Add | | • Area | • Distribute |
| • More than | | • Times | |
| | | • Double | |

# Key Words 3 of 10

When we resort to teaching students to use keywords we take the emphasis away from what is happening in the problem and instead put the focus on identifying specific words.  We take away sense making!

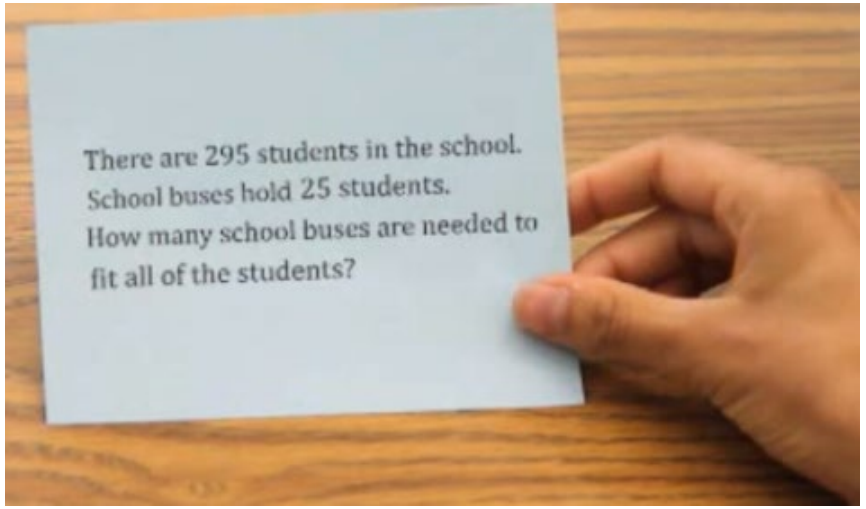
Examples:

- There are 24 tiles per box and I bought 7 boxes. How many tiles do I have altogether?
- Tonia had some pencils.  She gave 4 pencils to Wayne and then she had 9 pencils left.  How many pencils did Tonia have to start with?
- Aidan has 28 fish.  15 are blue and the rest are yellow.  How many are yellow?

# Key Words 4 of 10

Key Words Strategy Example

This is an example of an older elementary student. This type of instruction begins early and does not foster true understanding or prepare students for two-step and more advanced word problems.



# Key Words 5 of 10

Standard for Mathematical Practice 2:

Reason abstractly and quantitatively.

Students who 'make sense of quantities and their relationships in problem situations' are able to:

- Find the meaning of quantities.
- Represent a solution through pictures, numbers, and words

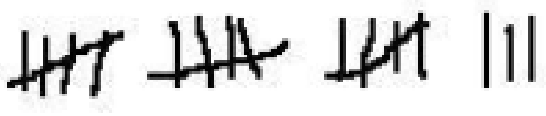
Sense Making

# Key Words 6 of 10

Alternatives to keywords...Sense Making!

Mrs. Harding has 3 children and they each have a box of chicken nuggets. Each box contains 6 chicken nuggets. How many chicken nuggets are there in all?

- How can students make sense of the situation?



6 + 6 + 6 = 18

6 × 3 = 18

# Key Words 7 of 10

Standard for Mathematical Practice 3:

Construct viable arguments and critique the reasoning of others.

Video 1:

Student is constructing an argument, discussing their thinking process aloud

Video 2:

Construct Viable Arguments & Critique the Reasoning of Others

# Key Words 8 of 10

Standard for Mathematical Practice 4:

Model with mathematics.

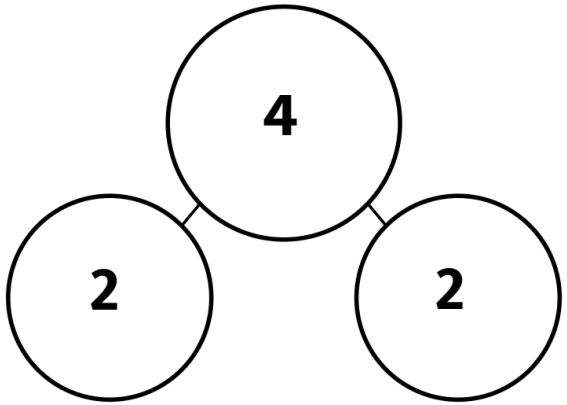
Students who 'model with mathematics' are able to:

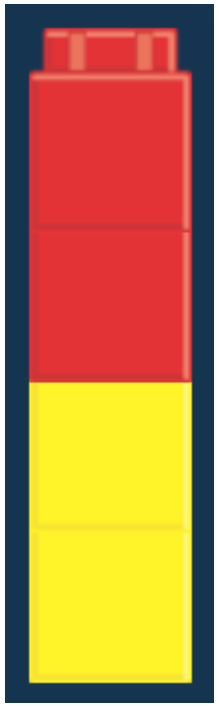
- convert text and mathematical concepts into multiple forms of representations with the use of concrete and representational models, symbols, and technology tools to demonstrate and communicate their mathematical thinking and reasoning.

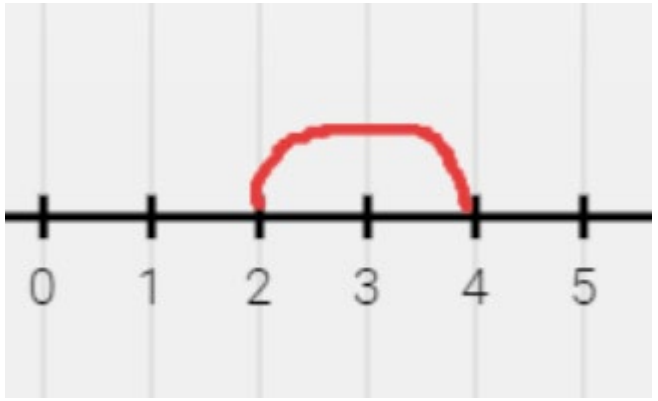
Rekenreks

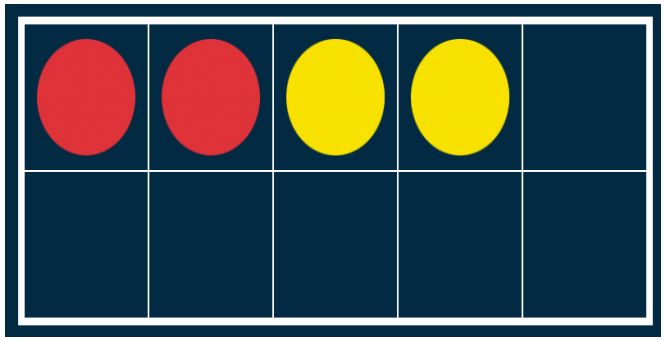
# Key Words 9 of 10

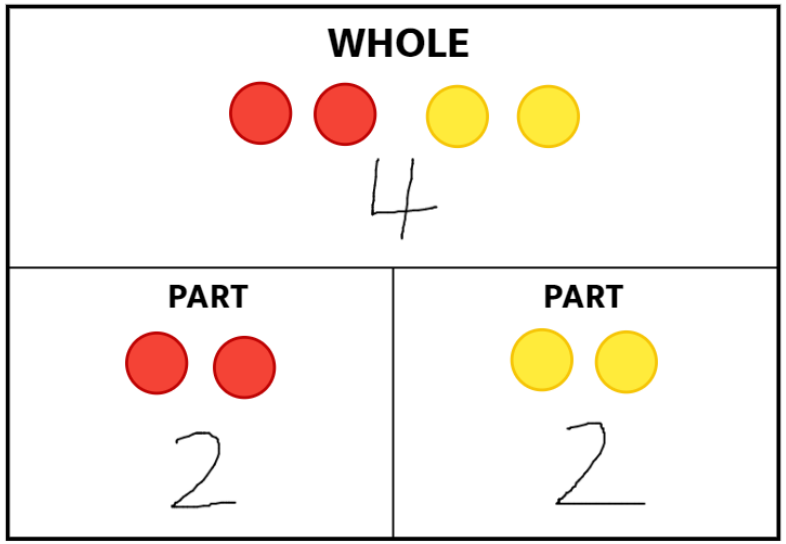
Models (examples):
Lisa had 2 red marbles and 2 yellow marbles. How many did she have altogether?
Concrete models
Pictorial models
Abstract models





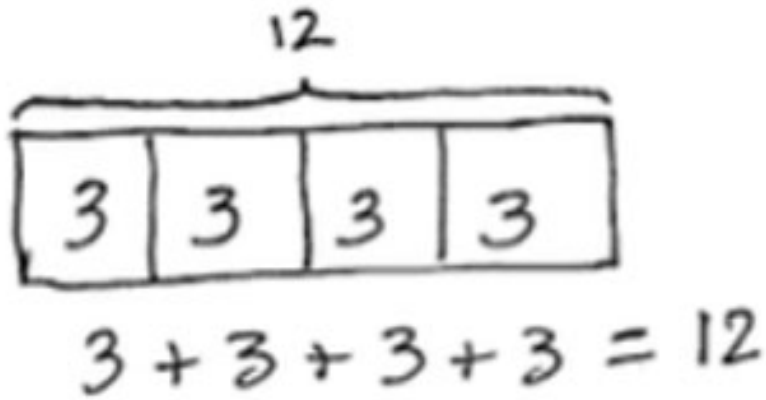


# Key Words 10 of 10

Models (another example):

Jacob has 12 oranges. He puts 2 oranges into each bag. How many bags does he have? Model the problem and label.

Tape Diagram



# Tools, Resources and Rich Tasks (1 of 13)

- Real Stories, Relevant Stories
- Deep Understanding
- Leveled
- Promote "Sense Making"
- Delay "answer getting"
- Focus on the process of the problem solving rather than the answer.
- Low floor, high ceiling tasks
- Teach questions, not answers

# Tools, Resources and Rich Tasks (2 of 13)

Problem of the Day

- Puppets, play-doh, stickers, toys, dot paints = fun!
- Use relevant situations and language that makes sense to your students
- Strategies Matter
- Delay ‘answer-getting’
- Perseverance matters
- Scaffolding matters

# Tools, Resources and Rich Tasks (3 of 13)

Planning Templates scaffold the problem solving process.

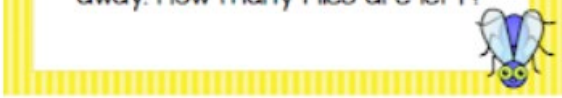
- Understand
- Plan
- Solve
- Check

| **Understand** (discuss, visualize, sketch, retell, translate) | **Plan** (which way will you solve) |
|---|---|
| | |
| **Solve** (carry out your plan) | **Check** (does your answer make sense? How do you know) |
| | |

# Tools, Resources and Rich Tasks (4 of 13)

**Read It**

We caught 9 flies but then 3 flew away. How many flies are left?



**Build It**

**Draw It**

# Tools, Resources and Rich Tasks (5 of 13)

Students are presented with the problem. Students begin to make sense of the problem and build it a concrete model. There is a lot of scaffolding at this point.

Intro-build it (Video)

Students solve the problem using concrete and pictorial representations. They share solution strategies. There is an abstract number sentence to model what was done.

Student writes equation (Video)

Students draws it to solve (Video)

# Tools, Resources and Rich Tasks (6 of 13)

3-act tasks

Act 1

- Provides a visual (photo or video, to hook the students into the task.
- Sparks curiosity and provokes  questions
- Little academic language
- Students “Notice and Wonder”
- Students determine a question (with teacher guidance)

Act 2

- Determine a question that needs to be answered.
- Make estimates.
- Determine what information is needed to solve

Act 3

- Construct a viable argument, share reflections
- Solve the problem
- Share solution strategies

# Tools, Resources and Rich Tasks (7 of 13)

3-Act Tasks : https://gfletchy.com/3-act-lessons/

Act 1 Visual:

Students are shown a photo or video that piques their interest and causes them to **notice** and **wonde**r.

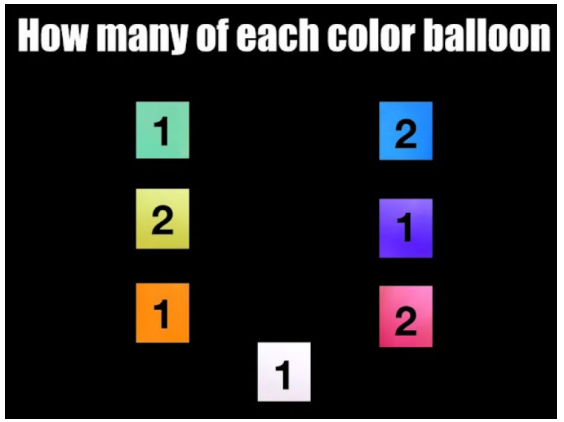
Act 1 (continued) Notice and Wonder:

Students discuss what they **notice** and what they **wonder** with peers.

Act 2:

Using some clues, students determine how many balloons were there.

Act 3: Solution



# Tools, Resources and Rich Tasks (8 of 13)

Another example…

Act 1:

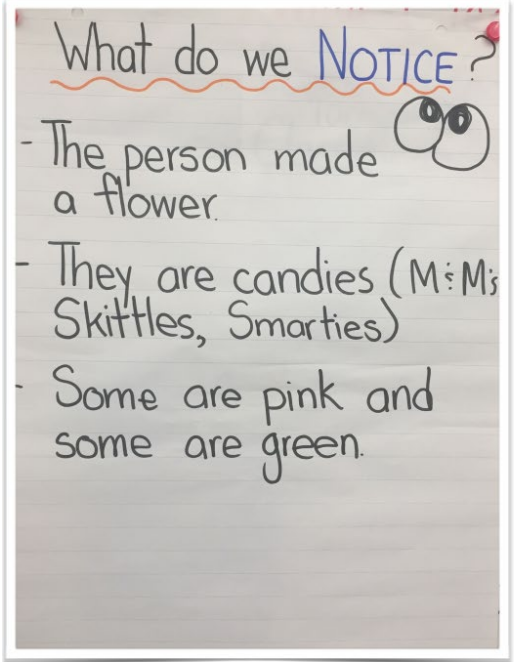
Students are shown realia, a video or a picture that piques their interest and causes them to **notice** and **wonder**. Using some clues, students determine a question.

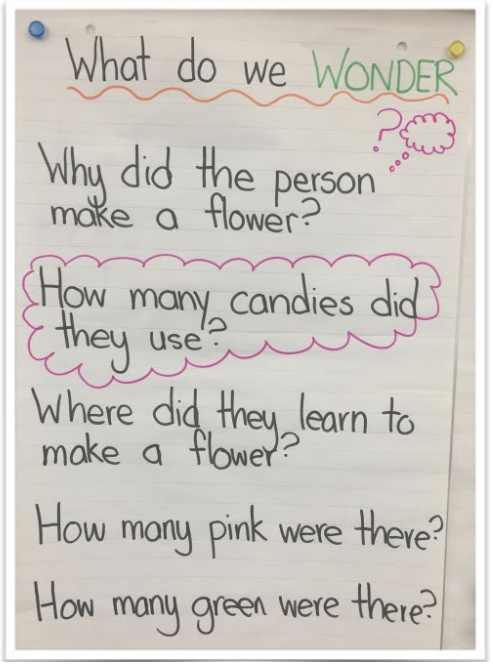
Act 2:

Students identify the information they will need to solve the problem. Students work to answer the question.

Act 3:

Student share strategies. The teacher may compare and connect students' strategies. The answer is revealed.



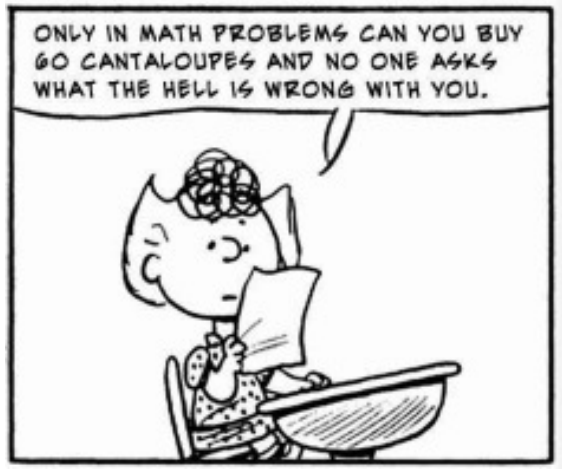
# Tools, Resources and Rich Tasks (9 of 13)

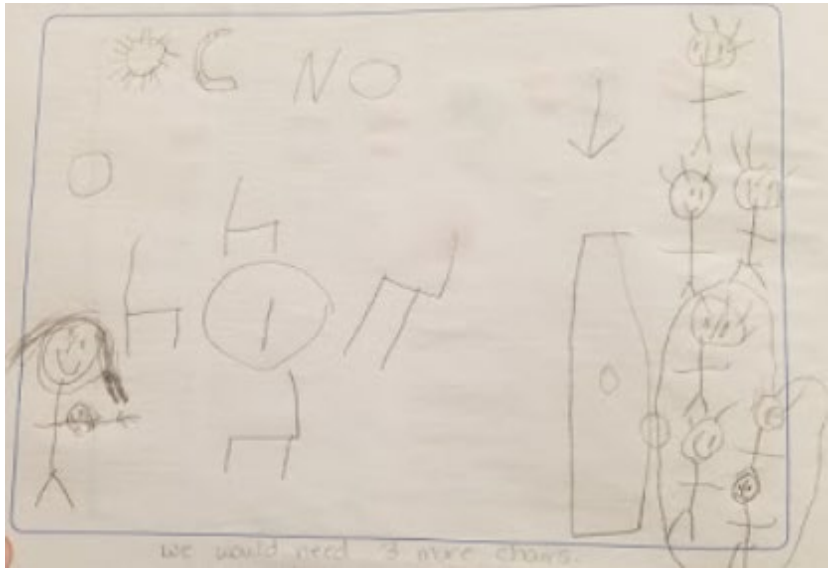
Present real world, rich tasks:

I am having a party. I set up 4 chairs at the table.

The doorbell rings, and there are 7 friends at the door.

Do I have enough chairs?

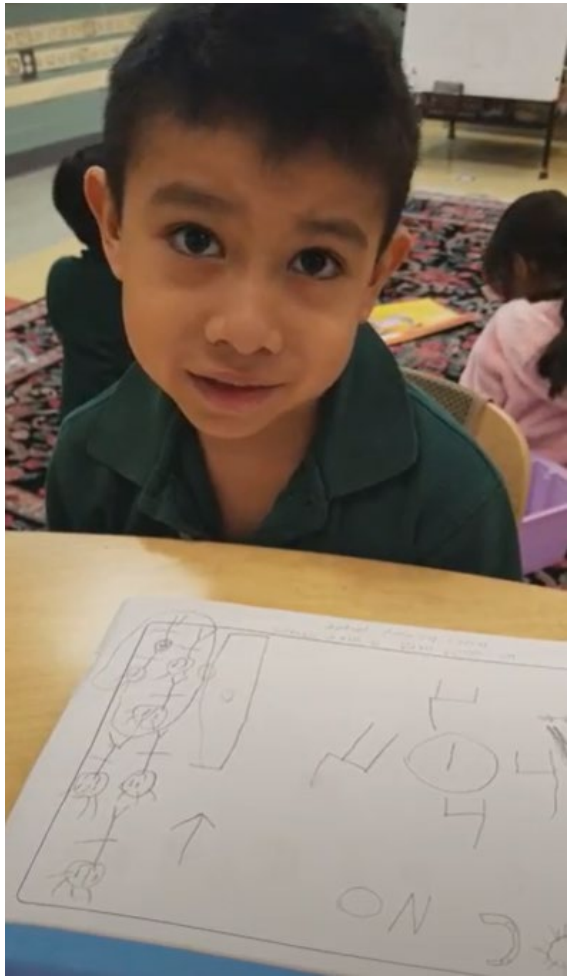


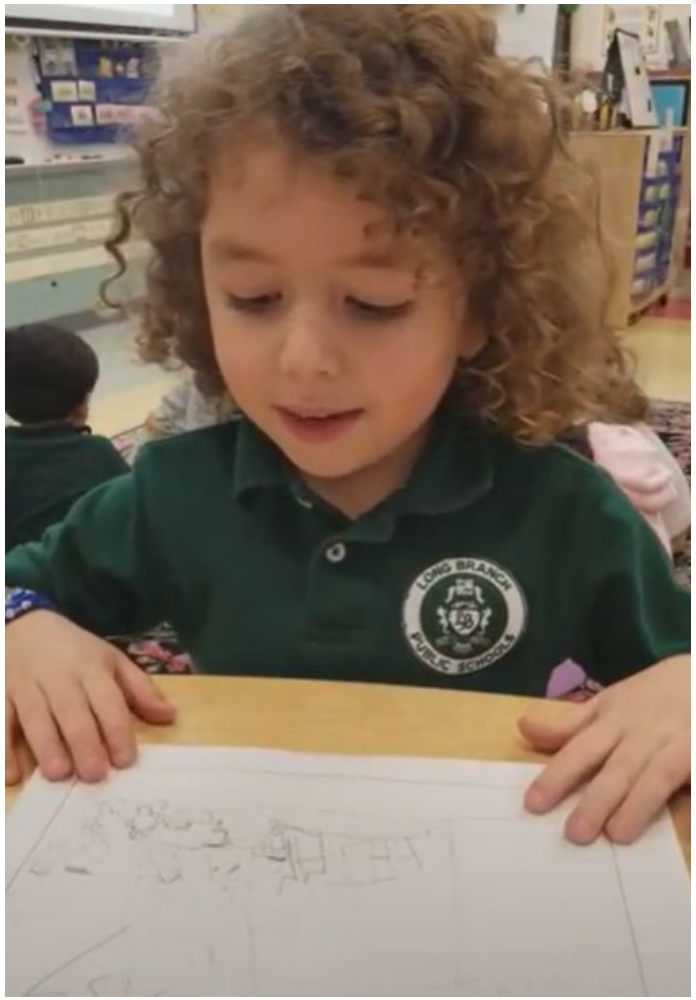



# Tools, Resources and Rich Tasks (10 of 13)

Solution Strategies for Rich Tasks:

- Solution strategy (video)
- Student solves and writes solution (video)





# Tools, Resources and Rich Tasks 11

## The Answer Is…


## Esti-mysteries

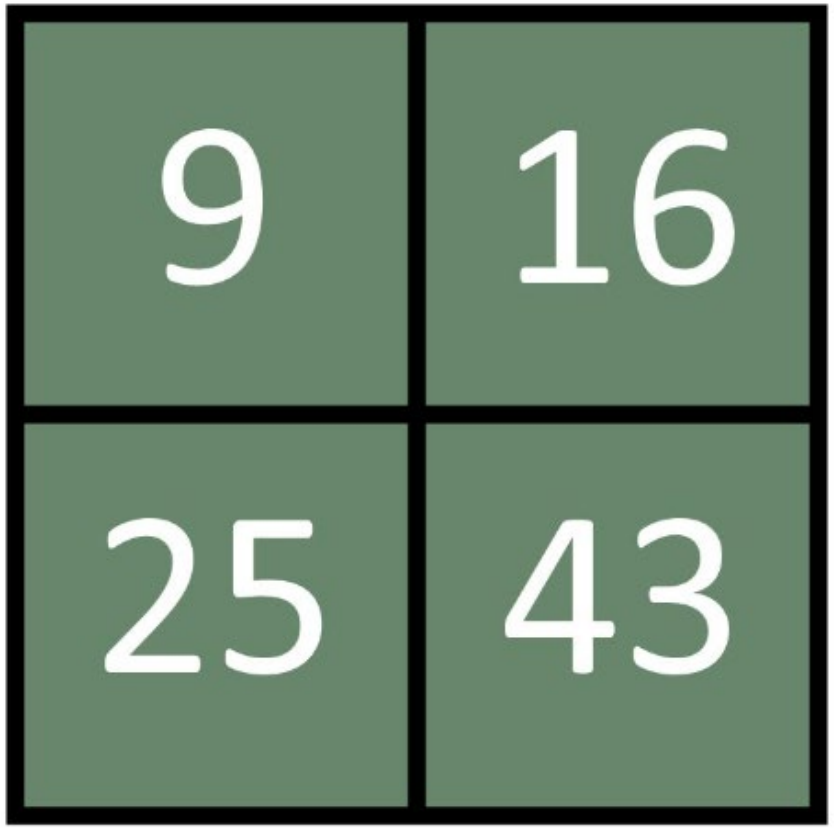
# Tools, Resources and Rich Tasks 12

- Numberless Word Problems
  - Example: Jeff has a pile of wood planks. He wants to use all of the planks to build coops for his chickens. Each coop will be the same size. How can he figure out how many planks to use for each coop? What information does he need to know?
- Student Created Problems

# Tools, Resources and Rich Tasks 13

[Which One Doesn't Belong]

The math practice standards ask students to construct arguments and critiquing the reasoning of others. It also requires teachers to attend to and listen closely to their students notice and wonder and have those a-ha moments in math!



# Closing

**What does it look like when students make sense of problems? (Video)**

# References and Resources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# Thank You!

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